Space Based Solar Power as a Contributor to Net Zero

Phase 2: Economic Feasibility - Annex A - Model Implementation

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ANNEX A – MODEL FORMULATION

This Annex provides a detailed description of the mathematical formulation of the model used to predict costs of space based solar power (SBSP) systems and the resulting levelized cost of electricity (LCOE). This annex is intended as a reference document, to ensure full transparency for any resultant LCOE predictions presented in the main body of the report. It provides detailed information on the model relationships which are embedded in the delivered spreadsheet. The model structure is displayed graphically at the start of the annex.

Within equations, expressions in $\mathrm{standard\ text}$ (as opposed to *italics*) refer to parameters defined in the tab "2. DefineRuns". The nomenclature for these parameters is tabulated at the end of this annex. The values of these parameters, and justification for their selection, are included in Annex B.

As the model is probabilistic, the majority of the inputs have been defined as distributions. Four distribution forms have been used: normal, uniform, log-uniform and triangular. A uniform distribution, between a lower bound l and an upper bound u is denoted:

$$X \sim U(l,u)$$

Uniform distributions are used where a range of values have been determined. A log uniform distribution, between a lower bound l and upper bound u is denoted:

$$X \sim L(l,u)$$

Log-uniform distributions are used in place of uniform distributions where the bounds span an order of magnitude or more.

Triangular distributions, with a lower bound l, an upper bound u and a peak or maximum likelihood m are denoted:

$$X \sim T(l,m,u)$$

Triangular distributions are used where a range of values is available, but either a clustering of values or a highly relevant source indicates one value is most likely.

Normal distributions, with mean μ and standard deviation σ are denoted:

$$X \sim N(\mu,\sigma)$$

Normal distributions are used where a single value is available, with a nominal uncertainty.

A.1 MODEL STRUCTURE

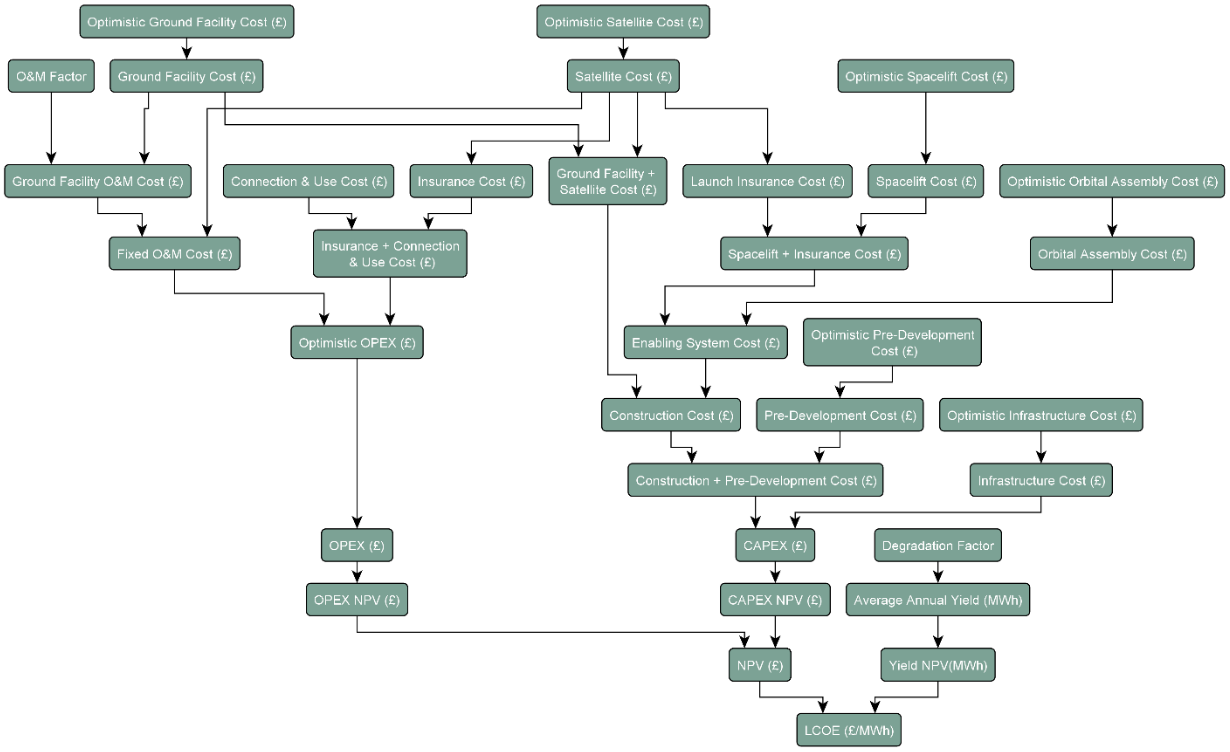

Figure A1: Model structure for LCOE calculation

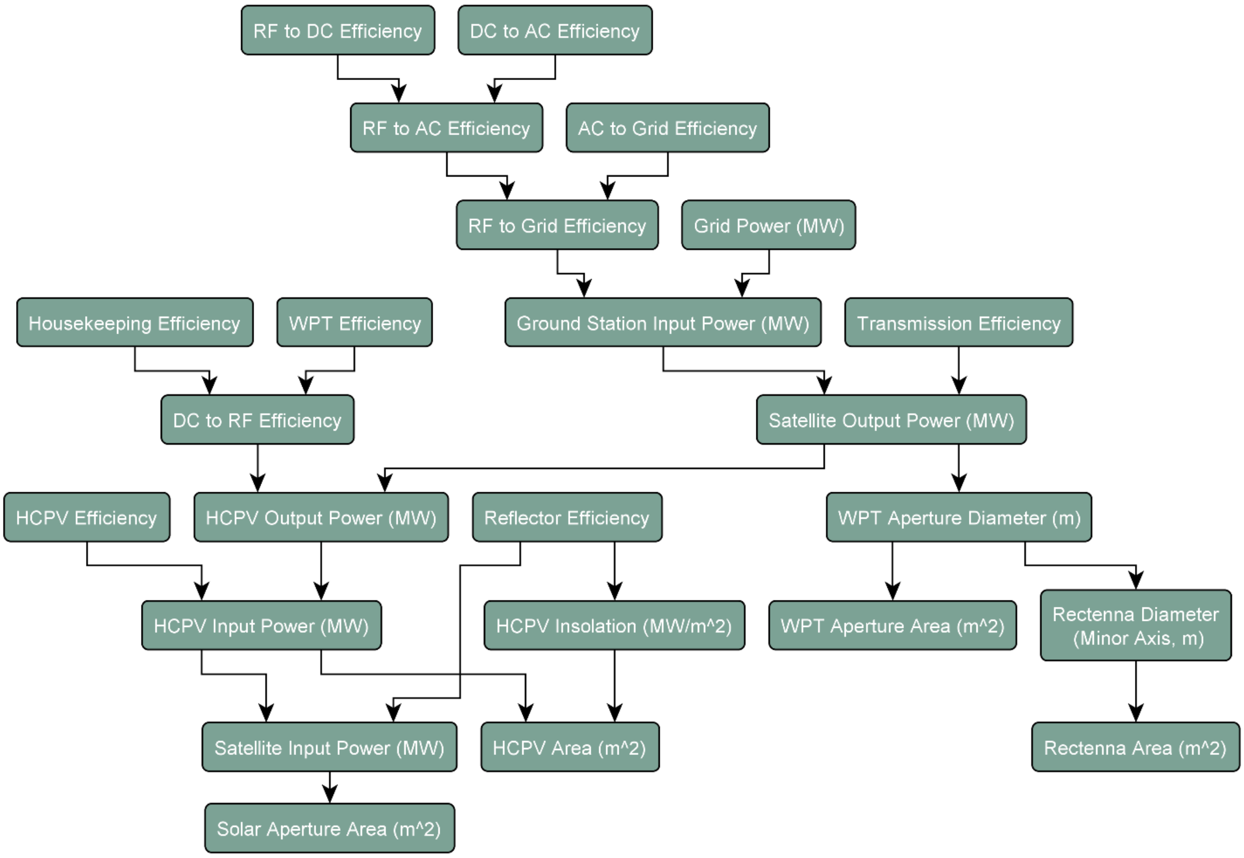

Figure A2: Model structure for system scale calculation

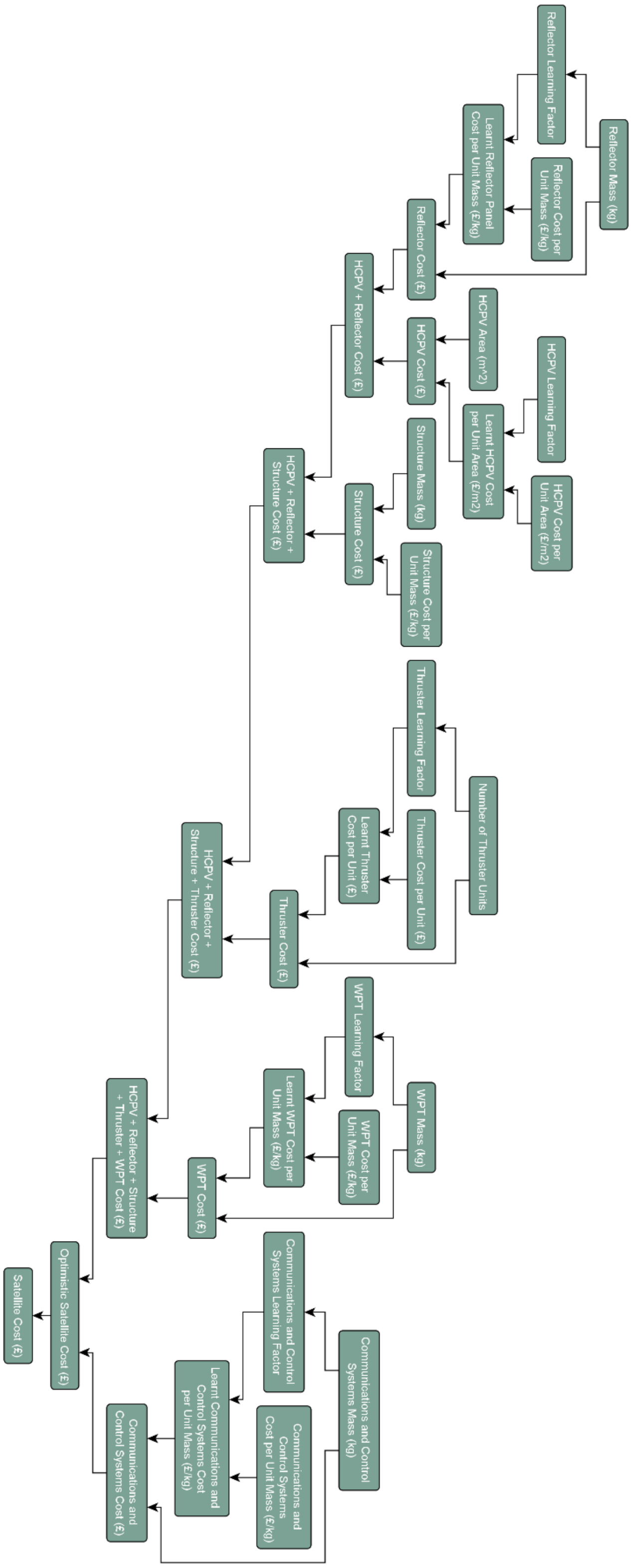

Figure A3: Model structure for satellite cost calculation

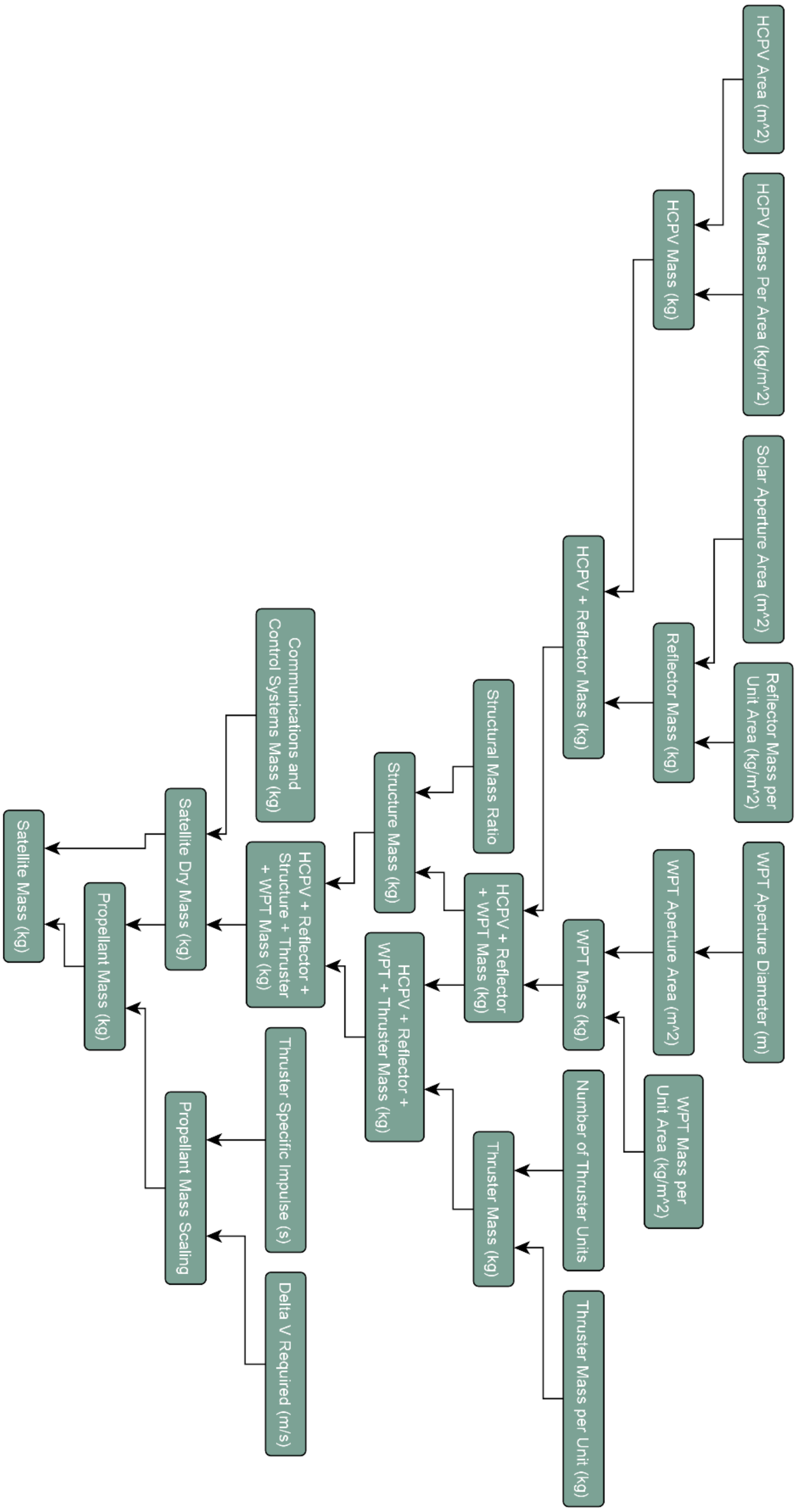

Figure A4: Model structure for satellite mass calculation

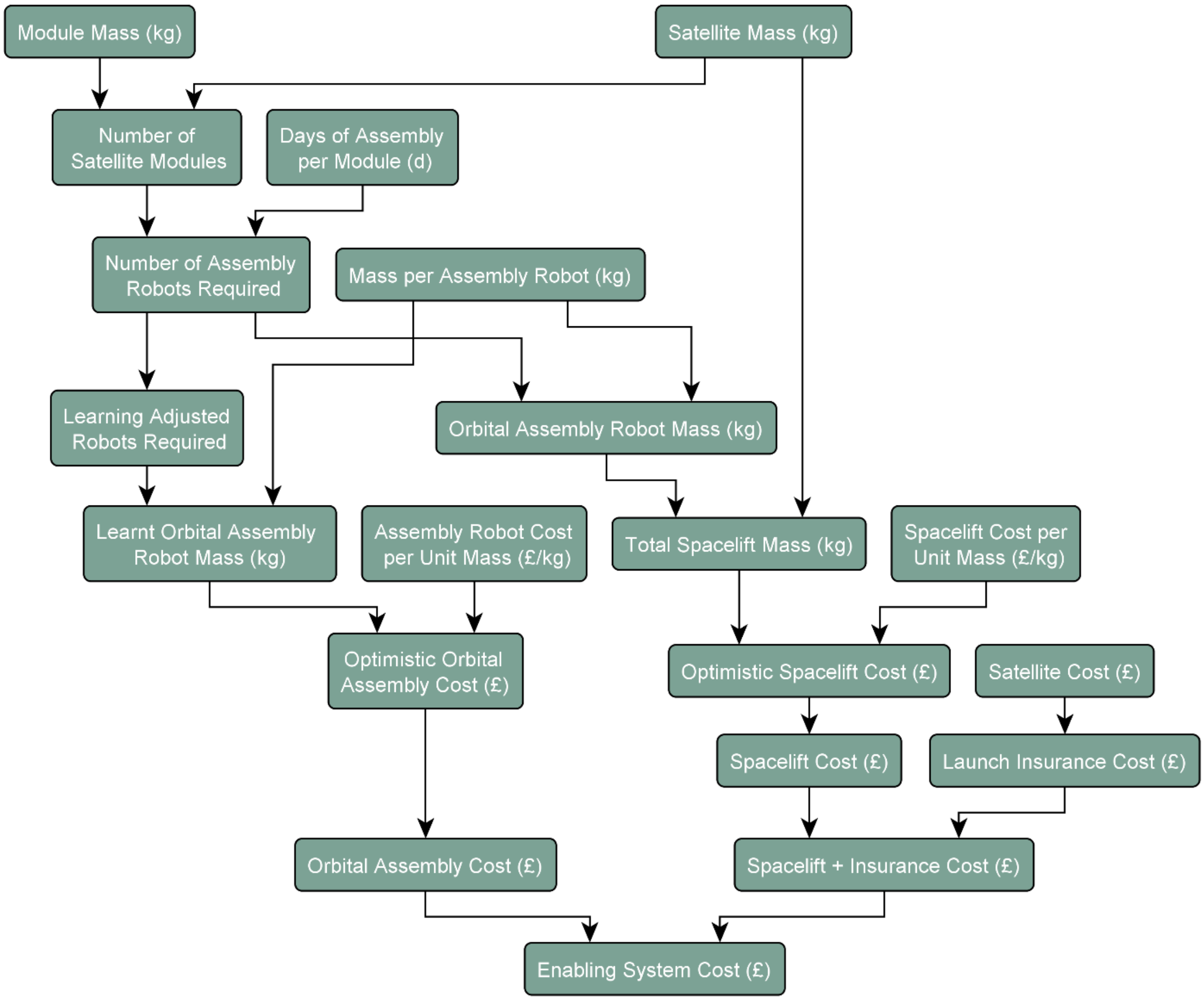

Figure A5: Model structure for enabling system cost calculation

A.2 LEVELISED COST OF ENERGY

The LCOE is the discounted lifetime cost of ownership and use of a generation asset, converted into an equivalent unit of cost of generation £/MWh for comparative purposes. In other words, it is the ratio of total costs associated with SBSP to the total amount of electricity expected to generate over its useful life. The figure is denominated in present value terms implying that the future cost outlays and electricity output are discounted to ensure consistency when comparing technologies which have different cost and benefit profiles over time. LCOE is defined as:

$$LCOE = \frac{N_{CAPEX} + N_{OPEX}}{Y}$$

Where:

- N_{CAPEX} is CAPEX net present value (£);
- N_{OPEX} is OPEX net present value (£);
- Y is system yield (MWh).

The net present values of CAPEX and OPEX are calculated by comparing cumulative discount rates:

$$N_{CAPEX} = \frac{CAPEX}{\mathrm{t}_{\mathrm{construction}}} D(\mathrm{t}_{\mathrm{construction}})$$

$$N_{OPEX} = \frac{OPEX}{\mathrm{t}_{\mathrm{operation}}} \left(D\left(\mathrm{t}_{\mathrm{operation}} + \mathrm{t}_{\mathrm{construction}}\right) - D(\mathrm{t}_{\mathrm{construction}}) \right)$$

Where:

- C_{CAPEX} is CAPEX cost (£);
- C_{OPEX} is OPEX cost (£);
- $D(t)$ is the cumulative discount factor at period t;

These relationships are derived from the sums of geometric series, based around the start of operation years. It is assumed that CAPEX and OPEX are incurred at constant rates throughout the construction and operation periods respectively, because it is not possible to determine a detailed activity timeline or profile of cost accrual at this early stage of development.

The yield is defined as:

$$Y = (\mathrm{P}_{\mathrm{grid}} \mathrm{f}_{\mathrm{load}} f_{degradation} * 24 * 365.25) \left(D\left(\mathrm{t}_{\mathrm{operation}} + \mathrm{t}_{\mathrm{construction}}\right) - D(\mathrm{t}_{\mathrm{construction}}) \right)$$

Where:

- $f_{degradation}$ is a factor to account for the failure and degradation of a limited number of modules during the life of the satellite. It is distributed according to:
  $$f_{degradation} \sim U\left(1 - \mathrm{f}_{\mathrm{deg,ub}} \mathrm{t}_{\mathrm{operation}}, 1 - \mathrm{f}_{\mathrm{deg,lb}} \mathrm{t}_{\mathrm{operation}}\right)$$
  This distribution assumes a range of average degradation rates from 0 to 0.5% per year;
- The factors 24 and 365.25 are to convert from years to hours.

A.3 OPEX

The OPEX is calculated as:

$$C_{OPEX} = C_{connection} + C_{operation} + C_{insurance}$$

Where:

- $C_{connection}$ is connection and use cost (£), distributed according to:

  $$C_{connection} \sim U\left(\mathrm{c}_{\mathrm{culb}} \mathrm{P}_{\mathrm{grid}} \mathrm{t}_{\mathrm{operation}}, \mathrm{c}_{\mathrm{cuub}} \mathrm{P}_{\mathrm{grid}} \mathrm{t}_{\mathrm{operation}}\right)$$

- $C_{operation}$ is operations and maintenance (O&M) cost (£), calculated with:

  $$C_{operation} = t_{operation}\left(f_{operation} C_{ground} + \mathrm{f}_{\mathrm{omlb}} C_{satellite}\right)$$

  Where $f_{operation} \sim T(\mathrm{f}_{\mathrm{omlb}}, \mathrm{f}_{\mathrm{omce}}, \mathrm{f}_{\mathrm{omub}})$ is a factor of construction cost incurred per year as an operational cost. The term $t_{operation} f_{operation} C_{ground}$ is the ground facility O&M cost.

  $C_{insurance}$ is insurance cost (excluding launch, £), given by:
  $$C_{insurance} = (1 + \mathrm{i}_{\mathrm{profit}}) \left(\mathrm{i}_{\mathrm{satyearone}} + \mathrm{i}_{\mathrm{satlife}}\left(\mathrm{t}_{\mathrm{operation}} - 1\right) \right) C_{satellite}$$

A.4 CAPEX

CAPEX is calculated as:

$$C_{CAPEX} = C_{construction} + C_{predev} + C_{infrastructure}$$

Where:

- $C_{construction}$ is construction cost (£), described in section A.6.
- C_{predev} is pre-development cost (£), given by:
$$C_{predev} \sim U\left(\mathrm{c_{predevlb}P_{grid}}, \mathrm{c_{predevub}P_{grid}}\right)$$
- $C_{infrastructure}$ is Infrastructure cost, given by:
$$C_{infrastructure} \sim U\left(\mathrm{c_{inflb}P_{grid}}, \mathrm{c_{infub}P_{grid}}\right)$$

A.5 POWER & EFFICIENCY

In order to produce cost and yield estimates for a SBSP system, its scale must be determined. The scale of each subsystem is modelled as dependent on the power which must be processed through that subsystem, and this power is, in turn, dependent on the subsystem efficiencies. This calculation leads to a hierarchy of systems, efficiencies and powers. The relationships do not *design* a SBSP system, but instead describe a design concept to allow the sensitivity to key parameters to be explored.

The radio-frequency (RF) power arriving at the ground station, P_{ground} (MW), is calculated as:

$$P_{ground} = \frac{P_{grid}}{\eta_{RF-grid}}$$

Where:

- $\eta_{RF-grid}$ is the RF-grid efficiency, given by:
$$\eta_{RF-grid} = \eta_{RF-DC}\eta_{DC-AC}\eta_{AC-grid}$$
- η_{RF-DC} is the conversion efficiency from RF to direct current (DC):
$$\eta_{RF-DC} \sim U(\mathrm{eta_{rfdclb}}, \mathrm{eta_{rfdcub}})$$
- η_{DC-AC} is the conversion efficiency from DC to alternating current (AC):
$$\eta_{DC-AC} \sim U(\mathrm{eta_{dcaclb}}, \mathrm{eta_{dcacub}})$$
- $\eta_{AC-grid}$ is the conversion efficiency from AC to the grid:
$$\eta_{AC-grid} \sim U\left(\mathrm{eta_{acgridlb}}, \mathrm{eta_{acgridub}}\right)$$

The RF power leaving the satellite is termed satellite output power, P_{So} (MW), and is calculated as:

$$P_{So} = \frac{P_{ground}}{\eta_T} = \frac{P_{grid}}{\eta_{RF-grid}\eta_T}$$

Where:

- η_T is the transmission efficiency, calculated from:
$$\eta_T = U(\mathrm{eta_{tlb}}, \mathrm{eta_{tub}})$$

The DC power emitted from the high-concentration photovoltaic (HCPV) module, P_{CPVo} (MW), is calculated as:

$$P_{CPVo} = \frac{P_{So}}{\eta_{DC-RF}} = \frac{P_{grid}}{\eta_{RF-grid}\eta_{T}\eta_{DC-RF}}$$

Where:

- η_{DC-RF} is the efficiency of the conversion from DC from the HCPV to RF emitted from the satellite, calculated as:
$$\eta_{DC-RF} = \eta_{housekeeping}\eta_{WPT}$$
- η_{WPT} is the efficiency of the wireless power transmitter (WPT), distributed according to $\eta_{WPT} \sim T(\mathrm{eta}_{\mathrm{wptlb}}, \mathrm{eta}_{\mathrm{wptml}}, \mathrm{eta}_{\mathrm{wptub}})$
- $\eta_{housekeeping}$ is a term to capture up to 10% additional power expenditure on satellite operation, distributed according to $\eta_{housekeeping} \sim (\mathrm{eta}_{\mathrm{hklb}}, \mathrm{eta}_{\mathrm{hkub}})$.

The power incident on the HCPV module, P_{CPVi} (MW), is calculated as:

$$P_{CPVi} = \frac{P_{CPVo}}{\eta_{CPV}} = \frac{P_{grid}}{\eta_{RF-grid}\eta_{T}\eta_{DC-RF}\eta_{CPV}}$$

Where:

- η_{CPV} is the HCPV module efficiency, distributed according to $\eta_{HCPV} \sim U(\mathrm{eta}_{\mathrm{hcpvlb}}, \mathrm{eta}_{\mathrm{hcpvub}})$. These values capture the efficiency of the PV and losses through the primary and secondary optical elements.

The power incident on the reflectors, termed satellite input power P_{si} (MW), is calculated as:

$$P_{Si} = \frac{P_{CPVi}}{\eta_{reflector}} = \frac{P_{grid}}{\eta_{reflector}\eta_{RF-grid}\eta_{T}\eta_{DC-RF}\eta_{CPV}}$$

Where:

- $\eta_{reflector} \sim U(\mathrm{eta}_{\mathrm{reflectlb}}, \mathrm{eta}_{\mathrm{reflectub}})$

A.6 CONSTRUCTION COST

The cost of construction, $C_{construction}$ (£), is calculated according to:

$$C_{construction} = C_{satellite} + C_{ground} + C_{enabling}$$

Where:

- $C_{satellite}$ is the cost of the satellite (£), described in section A.6.1;
- C_{ground} is the cost of the ground facility (£), described in section A.6.2;
- $C_{enabling}$ is the cost of enabling systems (£), described in section A.6.3.

A.6.1 SATELLITE COST

The cost of the satellite, $C_{satellite}$, is broken down into components as follows:

$$C_{satellite} = C_{reflector} + C_{CPV} + C_{thruster} + C_{CC} + C_{WPT} + C_{structure}$$

Where:

- $C_{reflector}$ is the cost of the reflector or mirror (£);
- C_{HCPV} is the cost of the HCPV modules (£);

- $C_{thruster}$ is the cost of thrusters required for station-keeping (£);
- C_{CC} is the cost of communications and control systems (£);
- C_{WPT} is the cost of the WPT modules (£);
- $C_{structure}$ is the cost of structural elements (£).

The mass of the satellite, $M_{satellite}$ (kg), is also calculated in order to inform space transportation costs, as follows:

$$M_{satellite} = M_{reflector} + M_{CPV} + M_{thruster} + M_{reaction} + M_{CC} + M_{WPT} + M_{structure}$$

Where:

- $M_{reflector}$ is the mass of the reflector or mirror, including its supporting structure (kg);
- M_{HCPV} is the mass of the HCPV modules, excluding supporting structure (kg);
- $M_{thruster}$ is the mass of thrusters required for station-keeping (kg);
- $M_{reaction}$ is the mass of propellant required for station-keeping (kg);
- M_{CC} is the mass of communications and control systems (kg);
- M_{WPT} is the mass of the WPT modules, excluding supporting structure (kg);
- $M_{structure}$ is the mass of structural elements (kg).

A.6.1.1 Reflector

The reflector scale is defined as the solar aperture area of the reflector, $A_{reflector}$, calculated as:

$$A_{reflector} = \frac{P_{Si}}{p_{solar}}$$

Note the physical area of the reflector will be larger than it's solar aperture due to the need to redirect the light. This discrepancy is accounted for in $m^{a}_{reflector}$. Where:

- p_{solar} is the power density from the sun,

The cost, $C_{reflector}$ and mass, $M_{reflector}$, of the reflector are factors of this area:

$$M_{reflector} = m^{a}_{reflector} A_{reflector}; \quad C_{reflector} = L_{reflector} c^{m}_{relfector} M_{reflector}$$

Where:

- $m^{a}_{reflector}$ is the mass per unit area of reflector (kg/m2) including supporting structure, distributed according to:
  $$m^{a}_{reflector} \sim T(\mathrm{ma}_{\mathrm{reflectlb}}, \mathrm{ma}_{\mathrm{reflectml}}, \mathrm{ma}_{\mathrm{reflectub}})$$
- $c^{m}_{reflector}$ is the cost per unit mass of the reflector (£/kg), distributed according to:
  $$c^{m}_{reflector} \sim U(\mathrm{cm}_{\mathrm{reflectlb}}, \mathrm{cm}_{\mathrm{reflectub}})$$
- $L_{reflector}$ is a learning factor for reflector manufacture, described in section A.6.4.

A.6.1.2 High Concentration Photovoltaic

The HCPV module scale is defined as the area of the primary optical element (POE), A_{HCPV} (m^2), defined as:

$$A_{HCPV} = \frac{P_{HCPVi}}{p_{HCPV}} = \frac{A_{reflector}}{f_{conc}}$$

Where:

- p_{HCPV} is the power density striking the HCPV module or HCPV insolation, calculated as:
$$p_{HCPV} = p_{solar} f_{conc} \eta_{reflector}$$
- f_{conc} is the reflector concentration factor, equal to two as a design assumption.

The mass, M_{HCPV} (kg), and cost, C_{HCPV} (£), of the HCPV module are calculated as:

$$M_{HCPV} = m^a_{HCPV} A_{HCPV}; \quad C_{HCPV} = L_{HCPV} c^a_{HCPV} A_{HCPV}$$

Where:

- m^a_{HCPV} is the mass per unit area of the HCPV module (kg/m²), distributed according to:
$$m^a_{HCPV} \sim T\left(\mathrm{ma_{hcpvlb}}, \mathrm{ma_{hcpvml}}, \mathrm{ma_{hcpvub}}\right)$$
- c^a_{HCPV} is the cost per unit area of the HCPV module (£/m2), distributed according to:
$$c^a_{HCPV} \sim T\left(c\mathrm{a_{hcpvlb}}, \mathrm{ca_{hcpvml}}, \mathrm{ca_{hcpvub}}\right)$$
- L_{HCPV} is a learning factor for HCPV manufacture, described in section A.6.4.

A.6.1.3 Thrusters

The scale of the solar electric thruster system required for station keeping is defined as a number of thrusters $N_{thruster}$, distributed according to:

$$N_{thruster} \sim N\left(\mathrm{n_{thrustmu}}, \mathrm{n_{thrustsigma}}\right);$$

The cost, $C_{thruster}$ (£), and mass $M_{thruster}$ (kg), of the thruster system are calculated as:
$$M_{thruster} = m^n_{thruster} N_{thruster}; \; C_{thruster} = L_{thruster} c^n_{thruster} N_{thruster}$$

Where:

- $m^n_{thruster}$ is the mass per thruster (kg), distributed according to:
$$m^n_{thruster} \sim U(\mathrm{m_{thrustlb}}, \mathrm{m_{thrustub}})$$
- $c^n_{thruster}$ is the cost per thruster (£), distributed according to:
$$c^n_{thruster} \sim U(\mathrm{c_{thrustlb}}, \mathrm{c_{thrustub}})$$
- $L_{thruster}$ is a learning factor for thruster manufacture, described in section A.6.4.

The reaction mass required, $M_{reaction}$ (kg), is calculated as:

$$M_{reaction} = M_{dry}\left(e^{\frac{\mathrm{deltaVt_{operation}} + \Delta V_{decom}}{g I_{thruster}}} - 1\right)$$

Where:

- M_{dry} is the "dry mass" of the satellite (kg) given by:
$$M_{satellite} = M_{reflector} + M_{CPV} + M_{thruster} + +M_{CC} + M_{WPT} + M_{structure}$$
- $I_{thruster}$ is the thruster specific impulse (s), distributed according to:
$$I_{thruster} \sim U(\mathrm{i_{thrustlb}}, \mathrm{i_{thrustub}})$$
- ΔV_{decom} is the delta-V required for decommissioning, distributed according to:
$$\Delta V_{decom} \sim U(\mathrm{dv_{decomlb}}, \mathrm{dv_{decomub}})$$

It is assumed that the cost of reaction mass is negligible compared to the cost of the thrusters.

A.6.1.4 Communications and Control

The cost of communications and control systems, C_{CC} (£), is calculated as:

$$C_{CC} = L_{CC} c^m_{CC} M_{CC}$$

Where:

- M_{CC} is the mass of communications and control systems, distributed according to:
$$M_{CC} \sim U(\mathrm{m_{cclb}}, \mathrm{m_{ccub}})$$
- c_{CC}^{m} is the cost per unit mass of communications and control systems (£/kg), distributed according to:
$$c_{CC}^{m} \sim T(\mathrm{cm_{cclb}}, \mathrm{cm_{ccml}}, \mathrm{cm_{ccub}})$$
- L_{CC} is a learning factor for communications and control systems manufacture, described in section A.6.4.

A.6.1.5 Wireless Power Transfer

The scale of the WPT system is defined by its aperture radius a_{WPT}, and its corresponding aperture diameter $2a_{WPT}$. This is calculated to match the assumed peak power density on the ground, according to:

$$2a_{WPT} = 2\sqrt{\frac{\mathrm{p_{ground}}\mathrm{a_{beam}^{2}}\left(\frac{3\times 10^{8}}{\mathrm{freq}}\right)^{2}}{P_{WPT}\pi}}$$

Note that this calculation does not include atmospheric absorption as it is assumed that the regulatory limit must be met when no absorption occurs. It also assumes a circular aperture, and no geometrical constraints on the transmitter due to the arrangement of satellite systems.

The cost, C_{WPT} (£) and mass, M_{WPT} (kg), of the WPT system are calculated as:

$$C_{WPT} = L_{WPT}c_{WPT}^{m}; \quad M_{WPT} = m_{WPT}^{a}\pi a_{WPT}^{2}$$

Where:

- c_{WPT}^{m} is the cost per unit mass of WPT (£/kg), distributed according to:
$$c_{WPT}^{a} \sim U(\mathrm{cm_{wptlb}}, \mathrm{cm_{wptub}})$$
- m_{WPT}^{a} is the mass per unit area of WPT (kg/m^2), distributed according to:
$$m_{WPT}^{a} \sim U(\mathrm{ma_{wptlb}}, \mathrm{ma_{wptub}})$$
- L_{WPT} is a learning factor for WPT manufacture, described in section A.6.4.

A.6.1.6 Structure

The mass of the structure, $M_{structure}$ (kg), is scaled on the mass of other elements:
$$M_{structure} = r_{structure}(M_{HCPV} + M_{WPT} + M_{reflector})$$

Where:

$r_{structure}$ is a factor distributed according to:
$$r_{structure} {\sim} N(\mathrm{r_{structmu}}, \mathrm{r_{structsigma}})$$

The cost of the structure, $C_{structre}$ (£), is calculated as:
$$C_{structure} = c_{structure}^{m}M_{structure}$$

Where $c_{structure}^{m}$ is the cost per unit mass of structure (£/kg), and is distributed according to:

$$c_{structure}^{m} \sim N(\mathrm{c_{structmu}}, \mathrm{c_{structsigma}})$$

No learning factor is included for structure manufacture due to the relatively low complexity and initial cost of structural modules.

A.6.2 GROUND FACILITY COST

The cost of the ground facility, C_{ground} (£), is decomposed as:

$$C_{ground} = C_{rectenna} + C_{land} + C_{control} + C_{BoP}$$

Where:

- $C_{rectenna}$ is the cost of the rectenna (£).
- C_{land} is the cost of land for the site (£).
- $C_{control}$ is the cost of power and mission control facilities (£), distributed according to:
  $$C_{control} \sim N(\mathrm{c}_{\mathrm{controlmu}}, \mathrm{c}_{\mathrm{controlsigma}})$$
- C_{BoP} is the cost of the electrical balance of plant (£), distributed according to:
  $$C_{BoP} \sim U\left(\mathrm{c}_{\mathrm{boplb}}\ \mathrm{P}_{\mathrm{grid}}, \mathrm{c}_{\mathrm{bopub}}\ \mathrm{P}_{\mathrm{grid}}\right)$$
  This distribution is derived from the range of estimates for terrestrial solar PV, as the balance of plant systems are expected to be similar. Reference [12] gives a cost range of £124,500-217,500/MW An EU assessment for solar PV [33] quotes £180,000/MW. For 100MW utility solar PV systems a cost of $0.13/W is quoted, leading to a value of £97,500/MW, used as a lower bound [34].

A.6.2.1 Rectenna and Land

The rectenna scale is defined by its minor axis radius, $a_{rectenna}$ (m), which is calculated as:

$$a_{rectenna} = \frac{1.22\left(\frac{3 \times 10^{8}}{\mathrm{freq}}\right)\mathrm{a}_{\mathrm{beam}}}{2a_{WPT}}$$

This calculation is based on the size of the WPT aperture and the diffraction pattern of the beam. A design assumption is that the rectenna is sized at the first minimum of the Airy disc which is a point of relatively diminishing returns from increasing rectenna size. It is possible that the size of the rectenna could be varied with consequences for the beam capture efficiency, $\eta_{capture}$.

The rectenna cost, $C_{rectenna}$ (£), is calculated as:

$$C_{rectenna} = \frac{c^{a}_{rectenna}\pi a^{2}_{rectenna}}{\sin(31°)}$$

Where:

- $c^{a}_{rectenna}$ is the rectenna cost per unit area (£/m2), distributed according to:
  $$c^{a}_{rectenna} \sim U(\mathrm{ca}_{\mathrm{rectlb}}, \mathrm{ca}_{\mathrm{rectub}})$$
- The factor of $\sin(31°)$ accounts for the inclination angle of the satellite, from a design assumption.

The cost of land, C_{land} (£), is calculated as:

$$C_{land} = \frac{c^{a}_{land}\pi a^{2}_{rectenna}}{\sin(31°)}$$

Where:

- c^{a}_{land} is the cost per unit area of land (£/m2), distributed according to:
  $$c^{a}_{land} \sim T(\mathrm{ca}_{\mathrm{landlb}}, \mathrm{ca}_{\mathrm{landml}}, \mathrm{ca}_{\mathrm{landub}})$$

A.6.3 ENABLING SYSTEMS COST

The cost of enabling systems, $C_{enabling}$ (£), is decomposed as follows:

$$C_{enabling} = C_{launchInsurance} + C_{spacelift} + C_{assembly}$$

Where:

- $C_{launchInsurance}$ is the cost of launch insurance (£), calculated as:
  $$C_{launchInsurance} = \mathrm{i}_{\mathrm{risk}} C_{satellite}$$
- $C_{assembly}$ is the cost of in-orbit assembly (£).
- $C_{spacelift}$ is the cost of transferring the satellite from earth to GEO (£)

A.6.3.1 Orbital Assembly

Orbital assembly is scaled on the number of robots required, N_{robot}, calculated from:

$$N_{robot} = \frac{t_{assembly} N_{module}}{t_{construction}}$$

- $t_{assembly}$ is the time taken to assemble one module and connect it to the satellite, assumed to be distributed according to:
  $$t_{assembly} \sim U\left(\mathrm{t}_{\mathrm{assemblylb}}, \mathrm{t}_{\mathrm{assemblyub}}\right)$$
- N_{module} is the number of satellite modules after all on-ground assembly, calculated as:
  $$N_{module} = \frac{M_{satellite}}{M_{module}}$$
- M_{module} is the mass of each module, distributed according to a design assumption:
  $$M_{module} \sim U(\mathrm{m}_{\mathrm{modulelb}}, \mathrm{m}_{\mathrm{moduleub}})$$

The cost, $C_{assembly}$ (£), and mass, $M_{assembly}$ (kg), of orbital assembly are then calculated as:

$$C_{assembly} = L_{robot} M_{assembly} c^{m}_{robot}; \; M_{assembly} = N_{robot} m^{n}_{robot}$$

Where:

- c^{n}_{robot} is the cost per robot (£), distributed according to:
  $$c^{m}_{robot} \sim U(\mathrm{cm}_{\mathrm{robotlb}}, \mathrm{cm}_{\mathrm{robotub}})$$
  m^{n}_{robot} is the mass per robot (kg), distributed according to:
  $$m^{n}_{robot} \sim T(\mathrm{m}_{\mathrm{robotlb}}, \mathrm{m}_{\mathrm{robotml}}, \mathrm{m}_{\mathrm{robotub}})$$
- L_{robot} is a learning factor for orbital assembly robot manufacture, described in section A.6.4.

To aid calculation of the cost of orbital assembly, the terms which depend upon the number of robots, L_{robot} and N_{robot}, are grouped. This involves the calculation of a “learning adjusted number of robots required”, N^{learn}_{robot} as an intermediate calculation step:

$$N^{learn}_{robot} = L_{robot} N_{robot}$$

The cost, $C_{assembly}$, is then calculated according to:

$$C_{assembly} = N^{learn}_{robot} m^{n}_{robot} c^{m}_{robot}$$

A.6.3.2 Spacelift

The cost of transferring the satellite from earth to GEO, $C_{spacelift}$ (£), calculated as:

$$C_{spacelift} = c^{m}_{spacelift}\left(M_{satellite} + M_{assembly}\right)$$

Where:

- $c^{m}_{spacelift}$ is the cost per unit mass transferred to GEO, distributed according to:

$$c_{spacelift} \sim L(\mathrm{cm}_{\mathrm{liftlb}}, \mathrm{cm}_{\mathrm{liftub}})$$

A.6.4 LEARNING FACTORS

The cost data available for many elements of the satellite, ground and enabling systems are for first of a kind of the respective systems. Significant cost savings are expected during mass production of these systems, as will be required for the modular SBSP satellite proposed. For the 'n of a kind' system considered, part of a 5-sattelite constellation, the unit costs of many modules will be greatly reduced compared to the cost data used.

The effects of mass production on costs are considered using learning factors [19]. The cost of the n^{th} identical module, C_n, (£) is calculated as:

$$C_n = C_1 n^{f_{LC}}$$

Where:

- C_1 is the cost of the first of a kind module (£)
- f_{LC} is a coefficient controlling the reduction, which is distributed according to:

$$f_{LC} \sim T(\mathrm{learn}_{\mathrm{elb}}, \mathrm{learn}_{\mathrm{eml}}, \mathrm{learn}_{\mathrm{eub}})$$

The total cost of N modules, assuming N_0 have previously been produced is the "n of a kind cost", C_{noak}:

$$C_{noak} = C_1 \sum_{i=N_0}^{N+N_0-1} i^{f_{LC}}$$

Where the top limit has been reduced by 1 to prevent over accounting due to the inclusive sum. The learning factor, L, is defined as the ratio of the "n of a kind" cost C_{noak}, to the cost of the equivalent first of a kind system, C_{foak}:

$$L = \frac{C_{noak}}{C_{foak}} = \frac{\sum_{x=N_0}^{N+N_0-1} x^{f_{LC}}}{\sum_{x=1}^{N} x^{f_{LC}}}$$

Evaluation of this sum is computationally challenging when N is large, hence for the purposes of modelling this is approximated using the inequality:

$$\sum_{x=N_0}^{N+N_0-1} x^{f_{LC}} \geq \frac{1}{N} \int_{N_0}^{N+N_0} x^{f_{LC}} dx$$

Therefore:

$$\sum_{x=N_0}^{N+N_0-1} x^{f_{LC}} \geq \frac{(N+N_0)^{f_{LC}+1} - (N_0)^{f_{LC}+1}}{(f_{LC}+1)}$$

$$\sum_{x=1}^{N} x^{f_{LC}} \geq \frac{(N+1)^{f_{LC}+1} - 1}{(f_{LC}+1)}$$

The learning factor is calculated assuming this approximation holds as an equality:

$$L = \frac{(N + N_0)^{f_{LC}+1} - (N_0)^{f_{LC}+1}}{(N + 1)^{f_{LC}+1} - 1}$$

The values of N for each of the modules which learning is applied to are either specified as parameters, such as $\mathrm{CC_N}$ for communications and control or are calculated as the mass of the system divided by a module mass, $\mathrm{m_{modulelearn}}$, for example for HCPV:

$$\mathrm{HCPV_N} = \frac{M_{HCPV}}{\mathrm{m_{modulelearn}}}$$

This module mass is distinct from that used in the assembly calculations, to account for the fact that some modules may undergo an initial assembly stage prior to spacelift. Values of N_0 are specified as parameters, such as $\mathrm{HCPV_{N0}}$ for HCPV.

A.7 PARAMETER NOMENCLATURE

Description	Type	Lower Bound / Standard Deviation	Maximum Likelihood / Mean / Constant Value	Upper Bound
OPEX Optimism Bias (%)	Constant		$\mathrm{ob}_{\mathrm{opex}}$	
Ground Station Optimism Bias (%)	Constant		$\mathrm{ob}_{\mathrm{ground}}$	
Satellite Optimism Bias (%)	Constant		$\mathrm{ob}_{\mathrm{satellite}}$	
Spacelift Optimism Bias (%)	Constant		$\mathrm{ob}_{\mathrm{spacelift}}$	
Orbital Assembly Optimism Bias (%)	Constant		$\mathrm{ob}_{\mathrm{assembly}}$	
At Grid Capacity (MW)	Constant		$\mathrm{P}_{\mathrm{grid}}$	
Design Life (y)	Constant		$\mathrm{t}_{\mathrm{operation}}$	
Discount Rate (Spend)	Constant		d	
Discount Rate (Yield)	Constant		d_{y}	
Construction Time (y)	Constant		$\mathrm{t}_{\mathrm{construction}}$	
Solar Insolation (W/m2)	Constant		$\mathrm{p}_{\mathrm{sun}}$	
Mirror Concentration Factor	Constant		$\mathrm{f}_{\mathrm{conc}}$	
RF Frequency (Hz)	Constant		freq	
Maximum Beam Distance (m)	Constant		$\mathrm{a}_{\mathrm{beam}}$	
RF Intensity Limit (W/m2)	Constant		$\mathrm{p}_{\mathrm{ground}}$	
Load Factor	Constant		$\mathrm{f}_{\mathrm{load}}$	
HCPV N0	Constant		$\mathrm{HCPV}_{\mathrm{N0}}$	
WPT N0	Constant		$\mathrm{WPT}_{\mathrm{N0}}$	
Thruster N0	Constant		$\mathrm{thruster}_{\mathrm{N0}}$	
Reflector N0	Constant		$\mathrm{reflector}_{\mathrm{N0}}$	
CC N	Constant		CC_{N}	
CC N0	Constant		$\mathrm{CC}_{\mathrm{N0}}$	
Orbit keeping delta V (m/s/y)	Constant		deltaV	
Gravitational Constant (m/s2)	Constant		g	
Rectenna N	Constant		$\mathrm{rectenna}_{\mathrm{N}}$	
Rectenna N0	Constant		$\mathrm{rectenna}_{\mathrm{N0}}$	
Learning Module Mass	Constant		$\mathrm{m}_{\mathrm{modulelearn}}$	
Launch Insurance Risk	Constant		$\mathrm{i}_{\mathrm{risk}}$	
Satellite Insurance Risk, First Year	Constant		$\mathrm{i}_{\mathrm{satyearone}}$	
Annual Satellite Insurance Risk, After First Year	Constant		$\mathrm{i}_{\mathrm{satlife}}$	
Insurance Profit Margin	Constant		$\mathrm{i}_{\mathrm{profit}}$	
Degradation Rate	Uniform	$\mathrm{f}_{\mathrm{deglb}}$		$\mathrm{f}_{\mathrm{degub}}$

Description	Type	Lower Bound / Standard Deviation	Maximum Likelihood / Mean / Constant Value	Upper Bound
O&M Factor	Triangle	f_{omlb}	f_{omml}	f_{omub}
Connection & Use Cost (£)	Uniform	c_{culb}		c_{cuub}
Infrastructure Cost (£)	Uniform	c_{inflb}		c_{infub}
Pre-Development Cost (£)	Uniform	$c_{predevlb}$		$c_{predevub}$
RF to DC Efficiency	Uniform	eta_{rfdclb}		eta_{rfdcub}
DC to AC Efficiency	Uniform	eta_{dcaclb}		eta_{dcacub}
AC to Grid Efficiency	Uniform	$eta_{acgridlb}$		$eta_{acgridub}$
Transmission Efficiency	Uniform	eta_{tlb}		eta_{tub}
WPT Efficiency	Triangle	eta_{wptlb}	eta_{wptml}	eta_{wptub}
Housekeeping Efficiency	Uniform	eta_{hklb}		eta_{hkub}
HCPV Efficiency	Uniform	eta_{hcpvlb}		eta_{hcpvub}
Reflector Efficiency	Uniform	$eta_{reflectlb}$		$eta_{reflectub}$
HCPV Mass Per Area (kg/m^2)	Triangle	ma_{hcpvlb}	ma_{hcpvml}	ma_{hcpvub}
Learning Exponent	Triangle	$learn_{elb}$	$learn_{eml}$	$learn_{eub}$
HCPV Cost per Unit Area (£/m2)	Triangle	ca_{hcpvlb}	ca_{hcpvml}	ca_{hcpvub}
Reflector Mass per Unit Area (kg/m^2)	Triangle	$ma_{reflectlb}$	$ma_{reflectml}$	$ma_{reflectub}$
Reflector Cost per Unit Mass (£/kg)	Uniform	$cm_{reflectlb}$		$cm_{reflectub}$
WPT Mass per Unit Area (kg/m^2)	Uniform	ma_{wptlb}		ma_{wptub}
WPT Cost per Unit Mass (£/kg)	Uniform	cm_{wptlb}		cm_{wptub}
Number of Thruster Units	Normal	$n_{thrustsigma}$	$n_{thrustmu}$	
Thruster Cost per Unit (£)	Uniform	$c_{thrustlb}$		$c_{thrustub}$
Thruster Mass per Unit (kg)	Uniform	$m_{thrustlb}$		$m_{thrustub}$
Communications and Control Systems Cost per Unit Mass (£/kg)	Triangle	cm_{cclb}	cm_{ccml}	cm_{ccub}
Communications and Control Systems Mass (kg)	Uniform	m_{cclb}		m_{ccub}
Structure Cost per Unit Mass (£/kg)	Normal	$c_{structsigma}$	$c_{structmu}$	
Structural Mass Ratio	Normal	$r_{structsigma}$	$r_{structmu}$	
Thruster Specific Impulse (s)	Uniform	$i_{thrustlb}$		$i_{thrustub}$
Land Cost per Unit Area (£/m^2)	Triangle	ca_{landlb}	ca_{landml}	ca_{landub}
Rectenna Cost per Unit Area (£/m^2)	Uniform	ca_{rectlb}		ca_{rectub}
Power Control + Mission Control Facility Cost (£)	Normal	$c_{controlsigma}$	$c_{controlmu}$	

Description	Type	Lower Bound / Standard Deviation	Maximum Likelihood / Mean / Constant Value	Upper Bound
Electrical Balance of Plant Cost (£)	Uniform	c_{boplb}		c_{bopub}
Spacelift Cost per Unit Mass (£/kg)	Log-uniform	cm_{liftlb}		cm_{liftub}
Orbital Module Mass (kg)	Uniform	$m_{modulelb}$		$m_{moduleub}$
Assembly Robot Cost per Unit Mass (£/kg)	Uniform	$cm_{robotlb}$		$cm_{robotub}$
Mass per Assembly Robot (kg)	Triangle	$m_{robotlb}$	$m_{robotml}$	$m_{robotub}$
Days of Assembly per Module (d)	Uniform	$t_{assemblylb}$		$t_{assemblyub}$
Decommissioning Delta V (m/s)	Uniform	$dv_{decomlb}$		$dv_{decomub}$

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